OPERACIONES CON VECTORES

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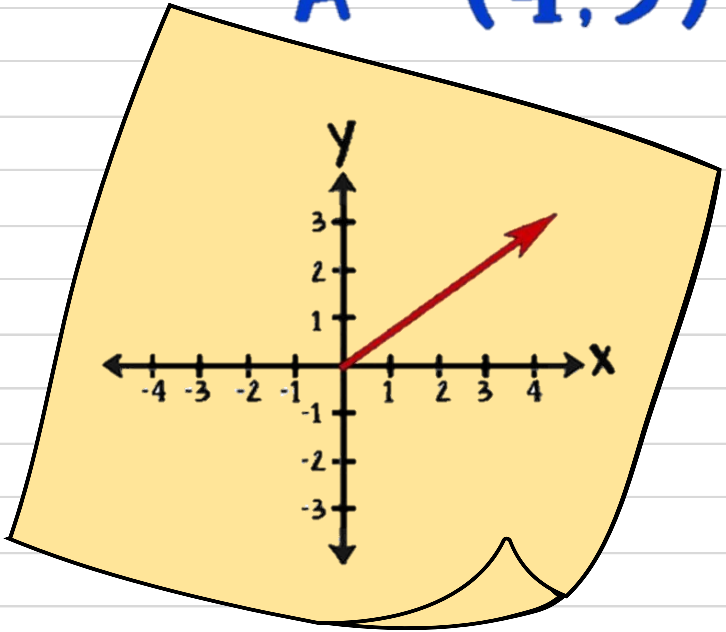
Contenido de la presentación

Vectores

Objeto geométrico
que tiene magnitud,
dirección y sentido

$\vec{A} = (4,3)$



OPERACIONES CON LOS VECTORES

02

SUMA DE VECTORES

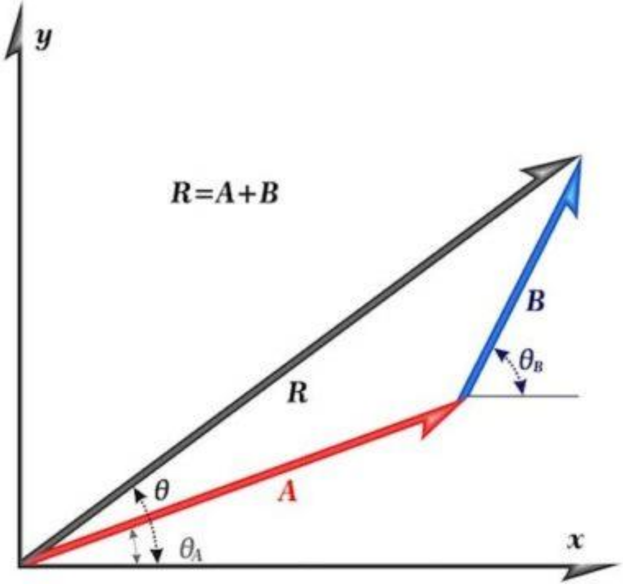
Suma de vectores

Dados los vectores A y B, la suma o resultante

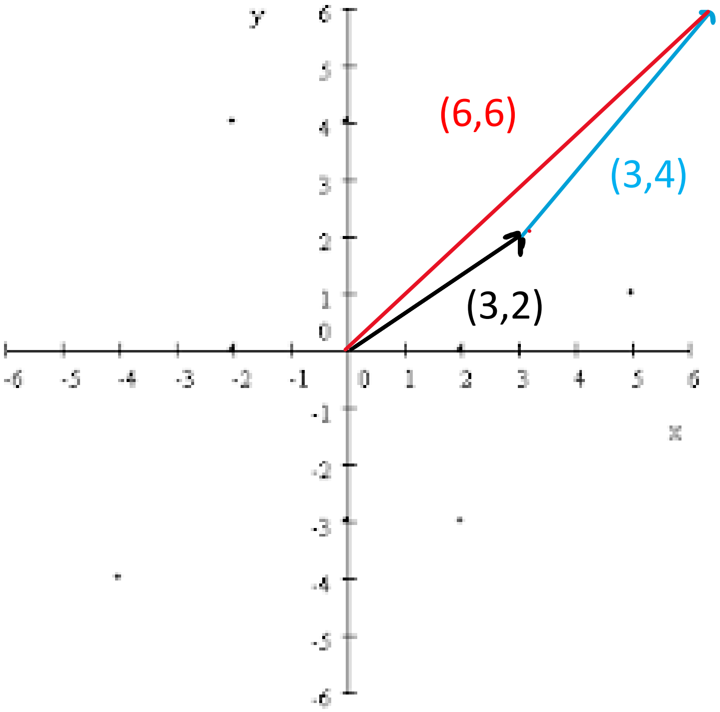
$$\bar{C} = \bar{a} + \bar{b}$$

Es aquel vector que resulta de unir el origen del vector sumado con el otro extremo, es decir:

$a^2 + b^2 = c^2$



Ejemplo suma

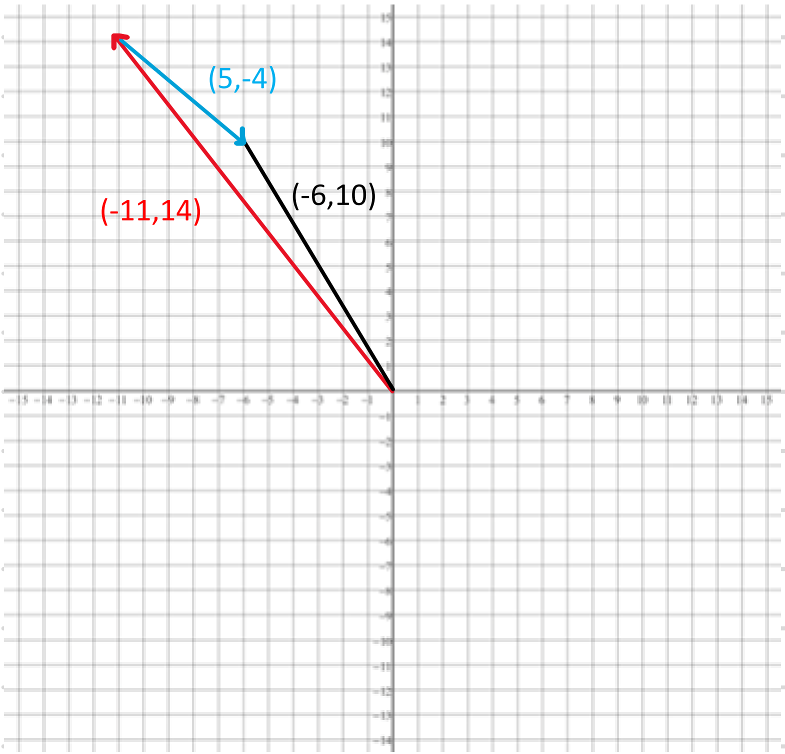


$$\begin{bmatrix}3\\2\end{bmatrix} + \begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}6\\6\end{bmatrix}$$

$$|c| = \sqrt{(6)^2 + (6)^2} = \sqrt{72} \approx 8.48$$

$$\theta = \tan^{-1}\left(\frac{6}{6}\right) = 45º$$

Ejemplo suma 2



$$\begin{bmatrix} -11 \\ 14 \end{bmatrix} + \begin{bmatrix} 5 \\ -4 \end{bmatrix} = \begin{bmatrix} -6 \\ 10 \end{bmatrix}$$

$$|c| = \sqrt{(-6)^2 + (10)^2} = \sqrt{360} \approx 18.97$$

$$\theta = \tan^{-1}\left(\frac{10}{-6}\right) = -59.03º$$

03

RESTA DE VECTORES

Resta de vectores

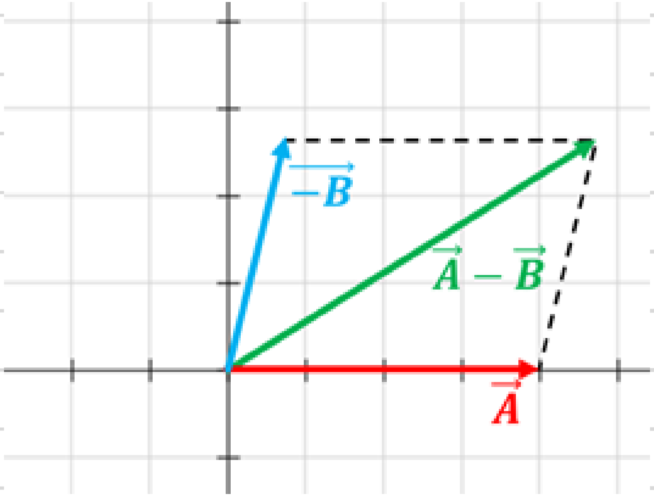
$a^2 + b^2 = c^2$

Dados los vectores A y B, la resta o resultante

$$\bar{C} = \bar{a} - \bar{b}$$

Es aquel vector que resulta de la suma de *a* con el opuesto de *b,* es decir:

$$\bar{a} - \bar{b} = \bar{a} + (-\bar{b})$$



Ejemplo resta

$$\mathbf{A} = \begin{bmatrix} 5 \\ 2 \\ 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -3 \\ 5 \\ 9 \end{bmatrix}$$

$$\mathbf{A} - \mathbf{B} = \begin{bmatrix} \mathbf{5} \\ \mathbf{2} \\ \mathbf{4} \end{bmatrix} - \begin{bmatrix} \mathbf{-3} \\ \mathbf{5} \\ \mathbf{9} \end{bmatrix} = \begin{bmatrix} \mathbf{5 - (-3)} \\ \mathbf{2 - 5} \\ \mathbf{4 - 9} \end{bmatrix}$$

$$\mathbf{A} - \mathbf{B} = \begin{bmatrix} 8 \\ -3 \\ -5 \end{bmatrix}$$

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} -3 \\ 5 \\ 9 \end{bmatrix} - \begin{bmatrix} 5 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -3 - 5 \\ 5 - 2 \\ 9 - 4 \end{bmatrix}$$

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} -8 \\ 3 \\ 5 \end{bmatrix}$$

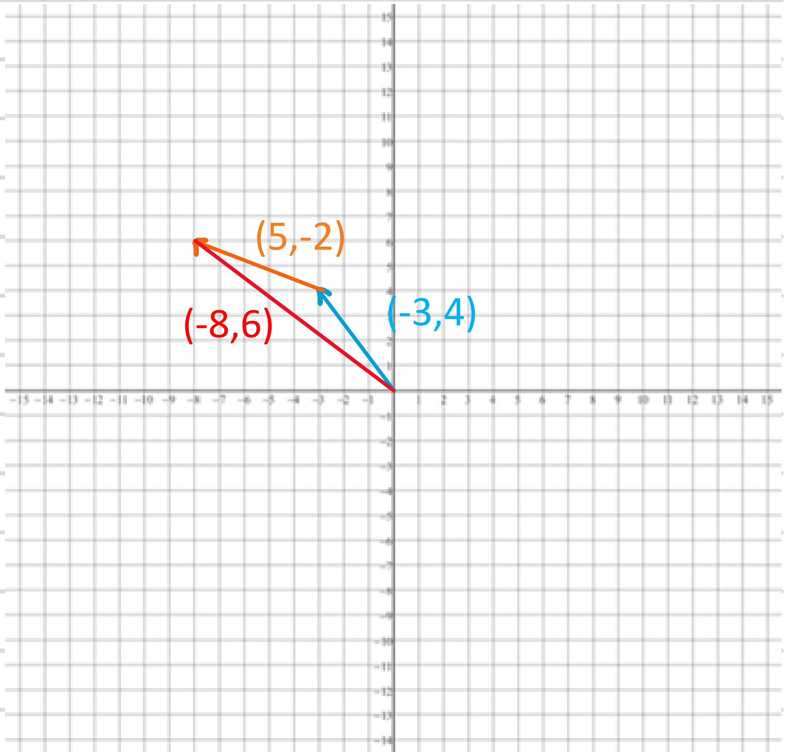
Ejemplo resta

$\bar{a} = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \qquad \bar{b} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} -3 \\ 4 \end{bmatrix} - \begin{bmatrix} 5 \\ -2 \end{bmatrix} = \begin{bmatrix} (-3) - (5) \\ (4) - (-2) \end{bmatrix} = \begin{bmatrix} -8 \\ 6 \end{bmatrix}$$

$$|c| = \sqrt{(-8)^2 + (6)^2} = \sqrt{100} = 10$$

$$\theta = \tan^{-1}\left(\frac{6}{-8}\right) = -36.86^{\circ}$$



04

PRODUCTO ESCALAR

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

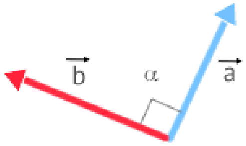
Producto punto o producto escalar

Dados dos vectores *a* y *b*, el producto escalar se define con un número real dado por:

$$\bar{a} \cdot \bar{b} = |a||b| \cos\theta$$

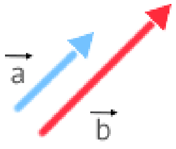
Donde θ es el ángulo formado entre ambos vectores.

Si $\vec{a}$ y $\vec{b}$ son **perpendiculares**
($\alpha = 90°$)



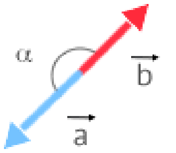
$\vec{a} \cdot \vec{b} = a \cdot b \cdot \cos(90) = 0$ (cos(90) = 0)

Si $\vec{a}$ y $\vec{b}$ son **paralelos** y con el **mismo sentido**
($\alpha = 0°$)



$\vec{a} \cdot \vec{b} = a \cdot b \cdot \cos(0) = a \cdot b$ (cos(0) = 1)

Si $\vec{a}$ y $\vec{b}$ son **paralelos** y con el **distinto sentido**
($\alpha = 180°$)



$\vec{a} \cdot \vec{b} = a \cdot b \cdot \cos(180) = - a \cdot b$ (cos(180) = −1)

Propiedades del prodcuto escalar

1. Conmutativa: $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$

2. Distributiva: $\bar{a}(\bar{b} + \bar{c}) = (\bar{a} \cdot \bar{b}) + (\bar{a} \cdot \bar{c})$

3. Asociativa: $p(\bar{a} \cdot \bar{b}) = (p\bar{a})\bar{b} = \bar{a}(p\bar{b})$

Ejemplo producto escalar

Sean:

$$\bar{a} = \begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix} \quad \bar{b} = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} \quad \bar{c} = \begin{bmatrix} 7 \\ 2 \\ 4 \end{bmatrix} \quad p = -5$$

Comprobar

1. Conmutativa: $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$

$$\begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} = [(-2)(4) + (5)(2) + (8)(6)] = -8 + 10 + 48 = 50$$

$$\begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix} = [(4)(-2) + (2)(5) + (6)(8)] = -8 + 10 + 48 = 50$$

Ejemplo producto escalar

Sean:

$$\bar{a} = \begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix} \qquad \bar{b} = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} \qquad \bar{c} = \begin{bmatrix} 7 \\ 2 \\ 4 \end{bmatrix} \qquad p = -5$$

Comprobar 2. Distributiva: $\bar{a}(\bar{b} + \bar{c}) = (\bar{a} \cdot \bar{b}) + (\bar{a} \cdot \bar{c})$

$$\begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix}\left(\begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 2 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix}\left(\begin{bmatrix} 4+7 \\ 2+2 \\ 6+4 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix}\left(\begin{bmatrix} 11 \\ 4 \\ 10 \end{bmatrix}\right) = [(-2)(11) + (5)(4) + (8)(10)] = -22 + 20 + 80 = 78$$

$$\left(\begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}\right) + \left(\begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 2 \\ 4 \end{bmatrix}\right) = [(-2)(4) + (5)(2) + (8)(6)] + [(-2)(7) + (5)(2) + (8)(4)]$$

$$[-8 + 10 + 48] + [-14 + 10 + 32] = [50] + [28] = 78$$

Ejemplo producto escalar

Recordar:

$$\begin{bmatrix}-2\\5\\8\end{bmatrix}\cdot\begin{bmatrix}4\\2\\6\end{bmatrix}=50$$

Sean:

$$\bar{a}=\begin{bmatrix}-2\\5\\8\end{bmatrix}\quad \bar{b}=\begin{bmatrix}4\\2\\6\end{bmatrix}\quad \bar{c}=\begin{bmatrix}7\\2\\4\end{bmatrix}\quad p=-5$$

Comprobar 3. Asociativa: $p(\bar{a}\cdot\bar{b})=(p\bar{a})\bar{b}=\bar{a}(p\bar{b})$

$$-5\left(\begin{bmatrix}-2\\5\\8\end{bmatrix}\cdot\begin{bmatrix}4\\2\\6\end{bmatrix}\right)=-5(50)=-250$$

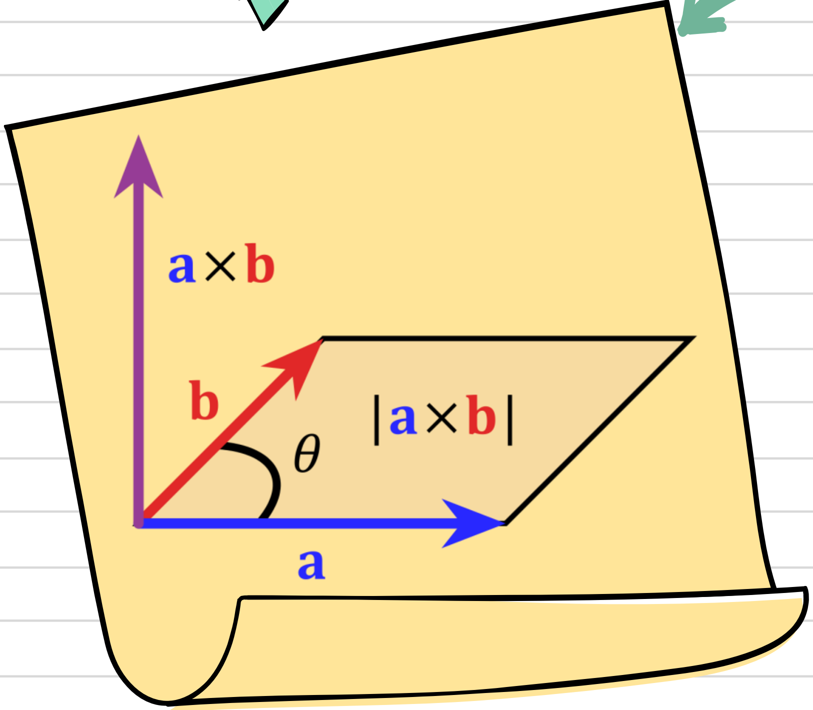
$$\left(-5\begin{bmatrix}-2\\5\\8\end{bmatrix}\right)\cdot\begin{bmatrix}4\\2\\6\end{bmatrix}=\left(\begin{bmatrix}(-5)(-2)\\(-5)(5)\\(-5)(8)\end{bmatrix}\right)\cdot\begin{bmatrix}4\\2\\6\end{bmatrix}=\begin{bmatrix}10\\-25\\-40\end{bmatrix}\cdot\begin{bmatrix}4\\2\\6\end{bmatrix}=[(4)(10)+(-25)(2)+(-40)(6)]=-250$$

$$\begin{bmatrix}-2\\5\\8\end{bmatrix}\cdot\left(-5\begin{bmatrix}4\\2\\6\end{bmatrix}\right)=\begin{bmatrix}-2\\5\\8\end{bmatrix}\cdot\left(\begin{bmatrix}(-5)(4)\\(-5)(2)\\(-5)(6)\end{bmatrix}\right)=\begin{bmatrix}-2\\5\\8\end{bmatrix}\cdot\begin{bmatrix}-20\\-10\\-30\end{bmatrix}=[(-2)(-20)+(5)(-10)+(8)(-30)=-250$$

05

PRODUCTO VECTORIAL

Producto vectorial



El producto cruz de dos vectores *a* y *b* es el producto *c*. Cuya longitud numericamente equivale al área del paralelogramo

Propiedades del prodcuto vectorial

1. Antionmutativa: $\bar{a}\ \times \bar{b} =\ \bar{b}\ \times \bar{a}$

2. Distributiva respecto a suma vectorial: $\bar{a} \times (\bar{b} + \bar{c}) = (\bar{a}\ \times \bar{b}) + (\bar{a}\ \times \bar{c})$

3. Asociativa respecto a producto escalar: $p(\bar{a}\ \times \bar{b}) = (p\bar{a})\ \times\ \bar{b} = \bar{a}\ \times (p\bar{b})$

4. No asociatividad: $\bar{a} \times (\bar{b}\ \times\ \bar{c})$

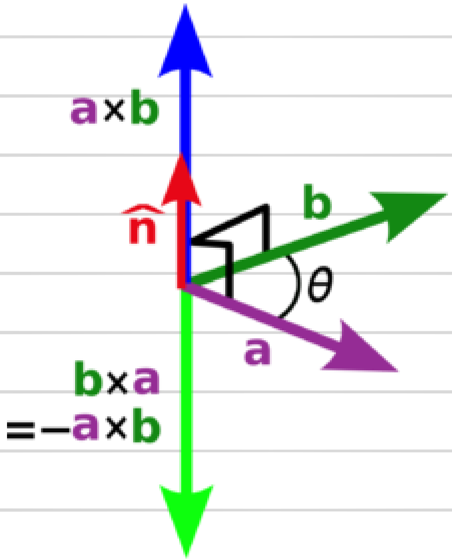
Producto cruz

$\hat{\imath}, \hat{\jmath}, \hat{k} \quad \rightarrow \quad Vectores\ unitarios$

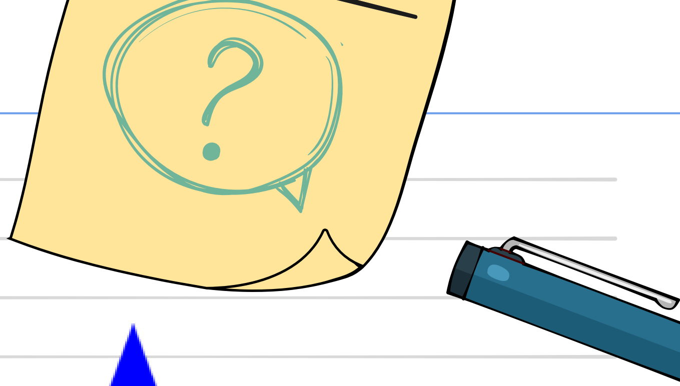
$\bar{a} = \{x_1, y_1, z_1\}$

$\bar{b} = \{x_2, y_2, z_2\}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = \hat{\imath}(y_1 z_2 - y_2 z_1) - \hat{\jmath}(x_1 z_2 - x_2 z_1) + \hat{k}(x_1 y_2 - x_2 y_1)$$



Ejemplo



$\hat{\imath}, \hat{\jmath}, \hat{k} \quad \rightarrow \quad$ *Vectores unitarios*

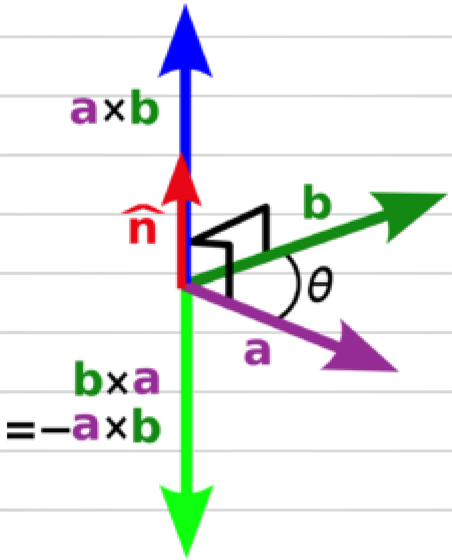
$$\bar{a} = \{4, 8, 12\}$$

$$\bar{b} = \{10, 3, -5\}$$

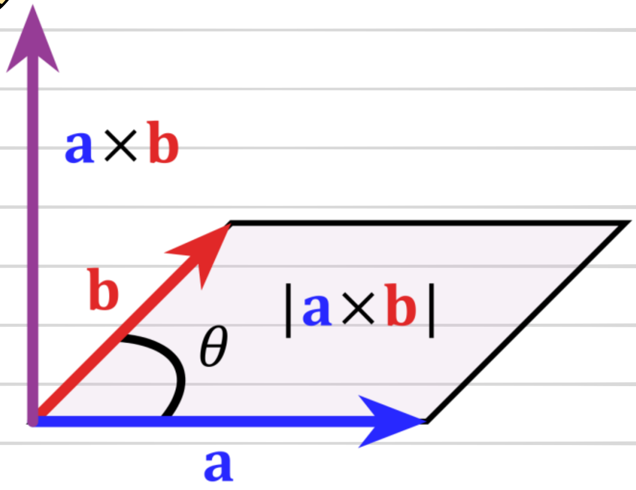
$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 8 & 12 \\ 10 & 3 & -5 \end{vmatrix} = \hat{\imath}[(8)(-5) - (3)(12)] - \hat{\jmath}[(4)(-5) - (10)(12)] + \hat{k}[(4)(3) - (10)(8)]$$

$$= \hat{\imath}[(-40) - (36)] - \hat{\jmath}[(-20) - (120)] + \hat{k}[(12) - (80)]$$

$$\bar{a} \times \bar{b} = -76\hat{\imath} + 140\hat{\jmath} - 68\hat{k}$$



Ejemplo



Obtener el área generada por los siguientes vectores

$$\bar{a} = \{7, -2, 5\} \qquad \bar{b} = \{1, 9, 3\}$$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 7 & -2 & 5 \\ 1 & 9 & 3 \end{vmatrix} = \hat{\imath}[(-2)(3) - (9)(5)] - \hat{\jmath}[(7)(3) - (1)(5)] + \hat{k}[(7)(9) - (1)(-2)]$$

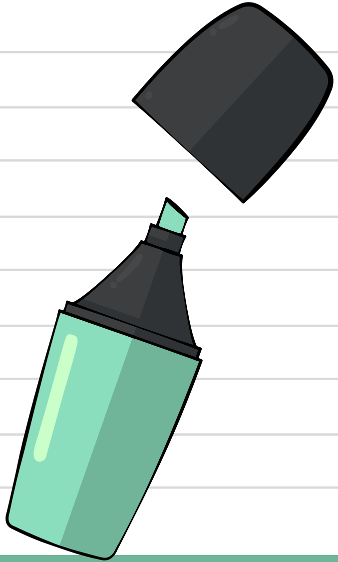
$$= \hat{\imath}[(-6) - 45] - \hat{\jmath}[(21) - (5)] + \hat{k}[(63) - (-2)]$$

$$\bar{a} \times \bar{b} = -51\hat{\imath} - 16\hat{\jmath} + 65\hat{k}$$

$$= \sqrt{(-51)^2 + (-16)^2 + (65)^2} = \sqrt{7082} \approx 84.15\ u^2$$

Gracias!

¿Tienen dudas?



6. Recursos

- Apuntes de cuaderno de 2021